MATH 147 QUIZ 7 SOLUTIONS

1. Let $D$ be the triangle in the $xy$-plane with vertices $(0,0),(1,1),(1,2)$. Find a linear transformation $T(u,v)=(au+bv,cu+dv)$ with $ad-bc\neq 0$ and $T(0,0)=0$, $T(1,0)=(1,1)$, $T(0,1)=(1,2)$. Set up but do not calculate the integral you would use to find the area of D. (5 Points)

   We begin by noting $T(1,0)=(a,c)$, and $T(0,1)=(b,d)$. Therefore, the conditions on where the vertices should go tells us $a=b=c=1$, and $d=2$. Thus, our linear transformation is $T(u,v)=(u+v,u+2v)$. To find out what the transformed integral would look like, we note that the Jacobian of T is

$$\det\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}=1.$$

   In addition, we note that the new region should be the triangle in the $uv$-plane with the vertices given above. Note that this is bounded by the lines $u=0$, $v=0$, $v=1-u$. Lastly, to find Area, we integrate the identity function. Thus, the area of $D$ is given by

$$\int_0^1\int_0^{1-u} dv\,du=\int_0^1\int_0^{1-v} du\,dv.$$

2. Let $R$ be the region in the first quadrant of the $xy$-plane bounded by the circles $4x^2+4y^2=1$ and $9x^2+9y^2=64$, and the line $x=y\sqrt{3}$. Find a rectangular region $R_0$ in the $r\theta$-plane and a transformation $G(r,\theta)$ taking $R_0$ to $R$. Graph $R_0$. (5 points)

   There is some ambiguity in which region exactly is $R$, but the process and answer will be similar for both. This will go through the region that is additionally bounded by $x=0$ (i.e. starting from the x-axis). First, we want the transformation $G(r,\theta)=(r\cos(\theta),r\sin(theta))$. Note the smaller circle is of radius $1/2$, while the larger one is of radius $8/3$. Thus, $R_0$ is bounded by $1/2\leq r\leq 8/3$. For the angle, note that we start at $\theta=0$, and go up to the line $x=\sqrt(3)y$. One can recognize this line passes through the unit circle at $x=\sqrt{3}/2, y=1/2$, or alternatively substitute in our transformation for x and y to solve, but no matter how we solve we should see that the angle is going up to $\pi/6$. Thus, the original rectangular region in $R_0$ is bounded by $0\leq\theta\leq\pi/6$ as well.

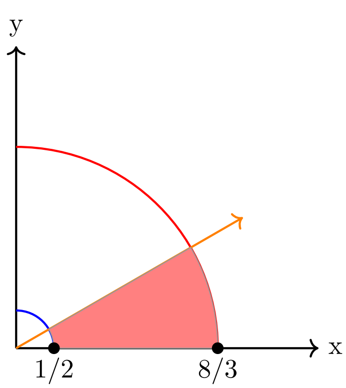


Figure 1: $R$

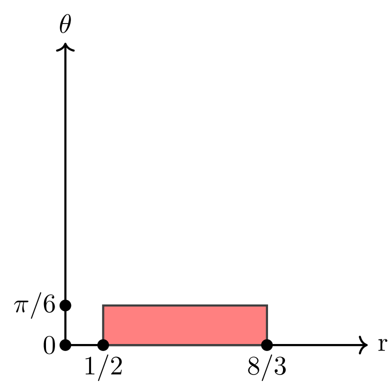


Figure 2: $R_0$